SQIPrime: A dimension 2 variant of SQISignHD with non-smooth challenge isogenies

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- SQIPrime does SQIsign2D using:
  - Non-smooth challenge isogenies.
  - Kani's Lemma.
  - Diagrams.

$$
\begin{tikzcd}
 & E_0 \arrow[rr, "{[\widehat{\phi_2\circ\psi}]_*\phi_1}"] \arrow[ddd, "\phi_2\circ\psi"] \arrow[ddr, "{[\widehat{\psi}]_*\phi_2}"] \arrow[dddrr, bend left, "\theta"] & & E_1' \\
E \arrow[ur, "\widehat{\psi}"] \arrow[ddr, "\phi_2"'] & & & \\
 & & E' & \\
 & E_1 \arrow[ur, "{[\phi_2]_*\widehat{\psi}}"] \arrow[rr, "\phi_1"'] & & E_0 \arrow[uuu, "{[\phi_1]_*\widehat{\phi_2\circ\psi}}"']
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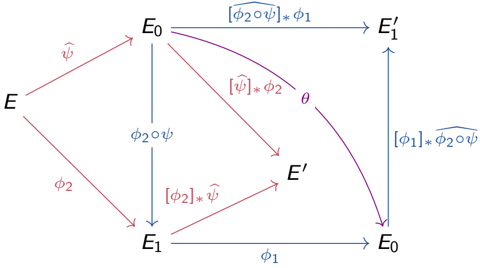

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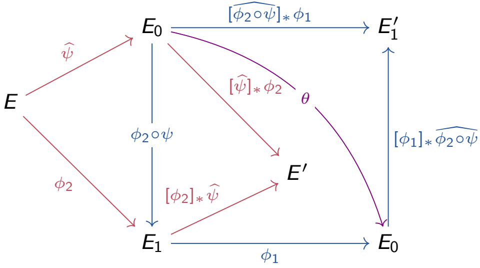

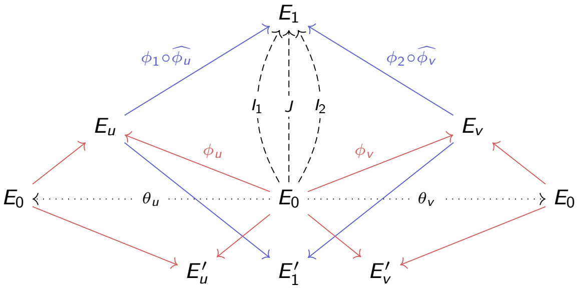

DeuringVRF Basis

- Finding I_ϕ is easy if $\deg(\phi)$ smooth, but what if $\deg(\phi)$ non-smooth ?

DeuringVRF Basis

A **DeuringVRF basis** (P, Q, ι, I_P) is:

- $P, Q \in E$ such that $\langle P, Q\rangle = E[q]$.
- $\iota \in \mathrm{End}(E)$ such that $\iota(P) = Q$.
- I_P is such that $I_P = I_\varphi$ with $\ker(\varphi) = \langle P\rangle$.

1. We can compute I_ϕ for $\deg(\phi) = q$.

$$\ker(\phi) = \langle [a]P + [b]Q\rangle \implies I_\phi = [a + b\iota]_* I_P$$

2. This is preserved through isogenies.

$$\ker(\phi) = \langle [a]\psi(P) + [b]\psi(Q)\rangle \implies I_\phi = [(a + b\iota)I_\psi]_* I_P$$

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Kani's Lemma

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E \arrow[r, "f"] \arrow[d, "g"'] & E_A \arrow[dl, "\theta"'] \arrow[d, dashed, "g'"] \\
E_B \arrow[r, dashed, "f'"'] & E_{AB}
\end{tikzcd}
$$

$$\deg(f) + \deg(g) = a + b = N$$
$$\gcd(a, b) = 1$$

$$\Longrightarrow$$

$$E_A \times E_B \xrightarrow{F := \begin{pmatrix} \widehat{f} & -\widehat{g} \\ g' & f' \end{pmatrix}} E \times E_{AB}$$

$$\ker(F) = \left\{ (f(P), -g(P)) \,\middle|\, P \in E[N] \right\}$$
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SQIPrime: KeyGen & Commitment

Public parameters : $p+1 = 2^e f$ with $q|p-1$ and $q \simeq 2^\lambda$ prime.
$+ (P, Q, \iota, I_P)$ a *DeuringVRF basis* of $E_0[q]$

- **Key Generation**:
  - Sample $\phi_{\text{sk}}, I_{\text{sk}}$ of degree $\ell_{\text{sk}} < 2^e$ prime.
  - $\binom{R}{S} = \mathbf{M} \cdot \phi_{\text{sk}} \binom{P}{Q}$.
  - $\text{pk} = E_{\text{pk}}, R, S \quad \text{sk} = \phi_{\text{sk}}, I_{\text{sk}}, \mathbf{M}$.

E_0

- **Commitment**:
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  - Challenge is $a \in \mathbb{Z}_q$.
  - $\ker(\phi_{\text{chal}}) = \langle C_a \rangle = \langle R + [a]S \rangle$.

- **Response**:
  - Retrieve $I_{\text{chal}} = [(b + c\iota) I_{\text{sk}}]_* I_P$.
  - Compute $I_{\text{res}} \sim I_{\text{chal}} I_{\text{sk}} I_{\text{com}}$ of norm $d \leq \sqrt{p}$.
  - Using $\text{End}(E_0)$ evaluate $\phi_{\text{chal+res}}(E_{\text{pk}}[2^e])$.
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SQIPrime: Verification idea

- **Verify**:
  - Using Kani's Lemma, represent $\phi_{\text{chal+res}}$.
  - Check $\phi_{\text{chal+res}}(C_a) \stackrel{?}{=} 0$.
  - Need an auxiliary isogeny ϕ_{aux} of degree $2^e - qd$.
- **Easy in dim 4**:
  - $2^e - qd = a_1^2 + a_2^2 \implies \phi_{\text{aux}} \in \text{End}(E_{\text{pk}}^2)$.
- **Harder in dim 2**:
  - What if $\deg(\phi_{\text{sk}}) = q$?

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SQIPrime: Auxiliary isogeny

- Need $\phi_{\text{aux}} : E_{\text{pk}} \to E_{\text{aux}}$ of degree $(2^e - qd)$.
- $\deg(\phi_{\text{sk}}) = q$.

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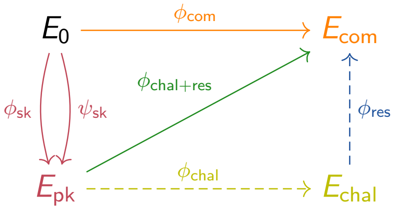

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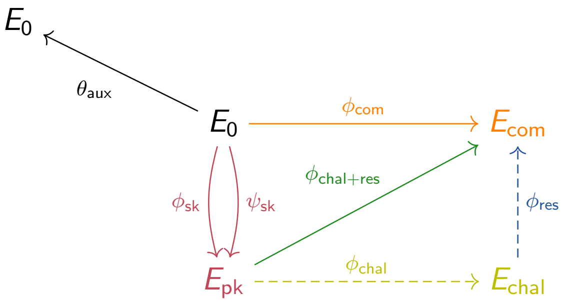

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$\deg(\theta_{\text{com}}) = d(2^e - qd)$

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- Need $\phi_{\mathsf{aux}} : E_{\mathsf{pk}} \to E_{\mathsf{aux}}$ of degree $(2^e - qd)$.
- $\deg(\phi_{\mathsf{sk}}) = q$.

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E_0 & E_0' \arrow[l, dashed, "\theta^2_{\mathsf{aux}}"'] & \\
 & E_0 \arrow[ul, "\theta_{\mathsf{aux}}"] \arrow[u, dashed, "\theta^1_{\mathsf{aux}}"'] \arrow[r, "\phi_{\mathsf{com}}"] \arrow[d, bend right, "\phi_{\mathsf{sk}}"'] \arrow[d, bend left, "\psi_{\mathsf{sk}}"] & E_{\mathsf{com}} \\
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▶ More subtle KeyGen and Verification.

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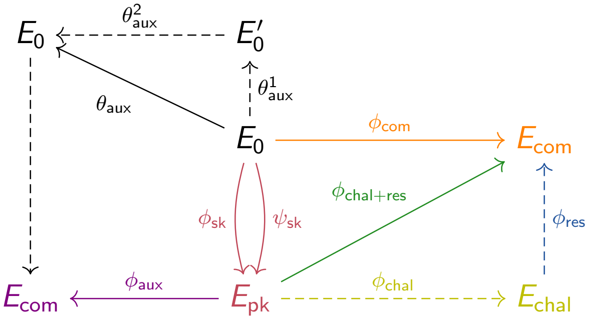

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- Given $\phi_{\text{chal+res}} \circ \widehat{\phi_{\text{aux}}}(E_{\text{aux}}[2^e])$, we compute F.
- Verify E_{pk} in codomain(F).
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$$

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Parameters

- **"SQIPrime"-primes**: p s.t. $p+1 = 2^e f$ with $q | p-1$ with $q \simeq 2^\lambda$ prime.
  - Can be found in polynomial time.

$p_{117}+1 = 2^{247} \cdot 79$	$q_{117} = 168118140144706967996895604212334429$
$p_{130}+1 = 2^{273} \cdot 19^2$	$q_{130} = (2^{136} \cdot 19 + 1)/955$
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SQIPrime as a Signature scheme

- Via Fiat-Shamir, SQIPrime is a quantum resistant signature scheme.

Scheme	λ	pk	signature	signature (compressed)
SQIPrime	128	191	320	299
	192	288	517	484
	256	384	635	600
SQISign	128	64	322	177
	192	92	-	267
	256	128	-	335
SQISignHD	128	64	208	109
	192	92	312	156
	256	128	416	208
Falcon	128	897	-	666
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Table: Size (in bytes) comparison between the different SQI-protocols.

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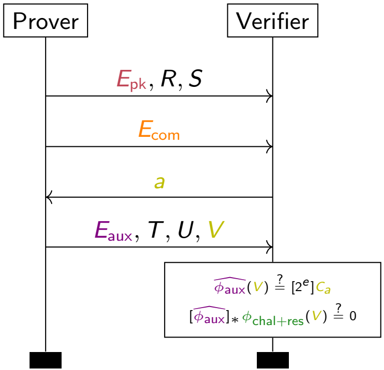
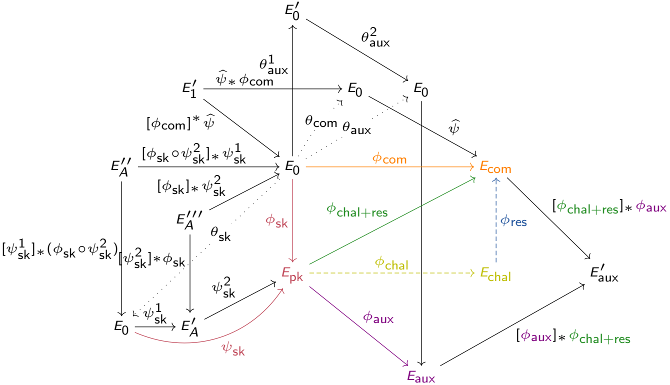


Efficiency

Scheme	prime	KeyGen	Signature	Verification
ApreSQI	p_{1973}	-	335000	-
	p_7	-	285000	-
	p_4	-	520000	-
SQISignHD	NIST-I	-	-	630
SQIPrime	p_{117}	473	677	205
	p_{130}	547	804	245
	p_{186}	950	1315	382
	p_{240}	1427	1927	564

Table: Computational times (in ms.) of the different SageMath implementation of SQI-signature schemes, measured on an Apple M1 CPU.



Paper

This is SQIPrime

Thank you !!

Code